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5. December 2007

MACROMODELS 2007
Warsaw

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Abstract. The paper presents the methodology for attaching probability distribution or intervals of variation to point forecasts. This methodology might prove significant for countries that have suffered deep structural transformations in their not very distant past. For these models, stability is more difficult to be achieved, because some coefficients lack accuracy of estimation, and this is not visible until intervals of variation are constructed. Forecasts consist traditionally in sets of values of key economic indicators, with no information regarding the associated uncertainty. Our assessment is that policy makers would benefit if they would be given probabilities as well as values, and the methodology of stochastic simulation, presented in this paper quantifies the uncertainty of the coefficients of the behavioural equations, on a reduced version of the Romanian Market Economy Model. In our paper we present the advantages of applying stochastic simulation on macromodels of emerging market economies, both from a cognitive and practical perspective. On one hand, researchers have an instrument to check the operational properties of a given model, and subsequently improve them, and, on the other hand, policy makers by incorporating the uncertainty into the decisional mechanism, have additional information which would help them in efficiently defining and promoting their targets.

JEL Classification System for Journal Articles: C12-C15, C52,
Key-words: macromodel, uncertainty, bootstrap, simulation

I. INTRODUCTORY CONSIDERATIONS

1. Forecasts consist traditionally of sets of values of key economic indicators for each specific scenario. The policy maker is then confronted with the problem of devising the policy mix which comes closer to his/her view of the future, based on the indicators provided by the model. Although the modelling tool is very useful, there is an inherent degree of uncertainty associated with the information it provides, uncertainty which is not acknowledged in most cases. Therefore there is an increase in accuracy of the forecast if information regarding values of the key economic indicators is given either as confidence intervals together with the associated probabilities, or as probability distributions. Our assessment is that policy makers would benefit if they would be given probabilities as well as values, and this is what the paper intends to do.

2. Our aim is to describe one way of attaching intervals of variation to main indicators, and probabilities instead of simple values. This methodology might prove significant for countries that have suffered deep structural transformations in their not very distant past, like Romania, and which have fractured, short data series on which to construct a model. The stability of the model for this type of economies is more difficult to be achieved, because some of the coefficients from the behavioural (stochastic) equations might be lacking in precision of estimation. If this happens, some indicators might be approximated with less accuracy than others, and this is not visible unless intervals of variation are constructed. Therefore, information regarding probabilities associated with different values can be of paramount importance.

3. We have exemplified the methodology devised on a reduced form of the macromodel of the Romanian Market Economy, Version 2005, a model elaborated for the National Commission for Prognosis under the PHARE programme.

3.1. This reduced form has 14 behavioural equations with a total of 43 coefficients. The chapter II of the paper gives a short presentation of the model and the subsequent simplification performed.

3.2. The methodology employed for our purpose – developed in the Chapter III – is known in the literature as stochastic simulation, and it is in general used for assessing the robustness properties of the model. In order to obtain intervals for the indicators we needed to run versions of the same model a sufficient number of times which would ensure enough values for the key indicators that would allow us to construct intervals. One of the most important sources of uncertainty of the model is represented by the behavioural equations, and their estimated coefficients, which have a direct impact on the predictions. Therefore, different sets of coefficients were generated, which were than introduced in the model and solved.

The first step is to construct sets of coefficients of the behavioural equations. The instrument which was employed is the bootstrap procedure. Starting from the data, standard errors, and estimates, with the help of bootstrapping, replicates of the original data base were created. For each replication, and each equation, estimates of the coefficients were computed with the help of OLS. By putting them together we have obtained the sample distribution for each coefficient. What we did further was that based on the coefficients' distributions we randomly generated around 500 sets of coefficients, which were later used to obtain solutions for the model

The paper describes how the bootstrap procedure helped with generating sets of coefficients, and looks at the specific properties of coefficients distribution.

3.3. Subsequently – especially in the chapter IV – there are presented the hypotheses which were made for the proposed exercise, and examines the results obtained by applying this procedure. More specifically it discusses the coefficient of variation and confidence intervals calculated for the main economic indicators.

3.4. The chapter V finalises the paper outlining some conclusions resulted from our work.

II. THE MACROMODEL USED IN SIMULATIONS

In order to facilitate computationally the simulations, a reduced form of the Romanian macromodel, Version 2005 (Dobrescu 2005, 2006) has been used. The main simplification results from the amputation of sectorial structure of economy (based on the input-output techniques), considered less significant for the present application.

1. The macromodel has been conceived with the purpose to estimate the short and medium–run implications of internal macroeconomic policies, one on hand, and of changes in international environment, on the other. Its reduced form is divided into five blocks:

- output,
- production factors and labour income,
- domestic absorption and foreign trade,
- prices and exchange rate, and
- financial and monetary variables.

2. The macromodel starts from the statistical data of previous years and several exogenous indicators, specific for the current year, which are separately obtained or extracted from other forecasts.

2.1. Among them, the expected index of disposable income plays a leading role.

2.2. The general consolidated budget is estimated using some fiscal and distributive coefficients, namely:

- ratio (to gross value added) of the value added tax, excises duties and other similar indirect taxes;
- ratio (to import of goods and services expressed in RON) of the custom duties;
- ratio (to gross domestic product) of the direct taxes and other revenues (excluding indirect taxes) of the general consolidated budget;

• ratio (to general consolidated budget expenditures) of the government transfers;
• ratio (to gross domestic product) of other expenditures (excluding government transfers) of the general consolidated budget;
• ratio (to general consolidated budget expenditures) of the budget subsidies on goods.

The general consolidated budget includes the state budget, the local budgets, the social insurance budget, and other similar funds; all of them exert income redistribution functions regulated by authorities.

2.3. The monetary policy is represented by the broad money (M2).

2.4. The international environment is characterised by the following parameters:
• net incomes and current transfers;
• foreign direct and portfolio investment;
• yearly index of world trade, volume;
• world trade deflator, special drawing rights (SDRs);
• short-term interest rate in advanced economies.

2.5. The number of population over 15 years – involved in the determination of labour force – is extracted from the current demographic projections.

2.6. The rate of tangible fixed assets depreciation is also set exogenously.

3. Regarding the behavioural relationships, the macromodel has retained those specifications which:
• are consistent with standard macroeconomic theorems;
• correctly reflect the peculiarities of the Romanian market economy;
• generate plausible results in simulations.

They refer to labour market, output, domestic absorption, foreign trade, prices, exchange rate, and interest rate.

3.1. In what concerns the interaction of labour supply-demand, three relationships have been selected: the labour force participation rate, the unemployment, and the nominal labour income per employed person.

3.1a. The labour force participation rate (prap) – as a ratio of labour force to population over 15 years – is defined depending on employment (E) in previous period. There were retained the first lag for prap and the second one for E, which reflect the relatively high inertia of the labour market processes. Therefore:

$$\text{prap} = f(\underset{(+)}{\text{prap}(-1)}, \underset{(+)}{E(-2)}) \qquad \text{(II.3.1.1)}$$

3.1b. Such a sluggishness is also present in the case of unemployment rate (ru). In addition, it appears to be mainly influenced by the evolution of unit labour cost (ULC), determined as a ratio between the labour income and the labour productivity. Consequently, the following specification has been adopted:

$$\text{ru} = f(\underset{(+)}{\text{ru}(-1)}, \underset{(+)}{\text{ULC}}) \qquad \text{(II.3.1.2)}$$

3.1c. With respect to the nominal labour income per employed person (LIE), two explicative factors seem to be essential: the unemployment rate (ru) and consumer price index (CPI). One lag is also involved:

$$\text{LIE} = f(\underset{(+)}{\text{LIE}(-1)}, \underset{(-)}{\text{ru}}, \underset{(+)}{\text{CPI}}) \qquad \text{(II.3.1.3)}$$

These relationships generate, also for the Romanian economy, standard slopes of the labour supply and labour demand (as a function of labour income per employed person).

3.2. The production function tries to combine the classical framework with the recent modelling approaches.

3.2a. The aggregate output (gross domestic product at constant prices) is estimated by an usual production function with capital, labour, and total factor productivity.

Since the series of tangible fixed assets was estimated using indirect methods, they are named "conventional tangible fixed assets". The capital is interpreted in its largest sense, including here not only technological equipments and direct productive buildings, but also infrastructure and other tangible fixed assets, taking into account that all of them influence the global economic performances. We maintain the assumption that the production function may include the real capital stock as such, without corrections derived from a disputable (and not clearly defined) normal utilization rate.

As in other similar approaches, the share of labour income in gross value added approximates the elasticity of output with respect to labour (the coefficient alpha). This was computed using the corresponding data of input-output tables (alpha1). There are, however, some reasons to think that alpha1 underestimates the real contribution of labour factor to output. One of them comes from the difficulties to evaluate this contribution in the so-called "unobservable" economy, including the production of households for self-consumption (relatively important in Romanian economy). This is why, a second source of data was utilised: the gross disposable income of households. Consequently, the used in macromodel alpha is named extended share of the labour income in gross value added.

3.2b. Two categories of determinants are involved in estimation of the total factor productivity:

- the level of alpha itself and, on the other hand,
- several variables, which decisively influence the technologies and the utilisation rate of the productive capacities.

3.2b1. Regarding alpha, it seems realistic to assume that:

- when actual alpha is less than its long-run (equilibrium) level, the labour force is not stimulated to reach the highest potential output;
- conversely, if alpha surpasses such an optimal level, the firms are obliged to restrain their activity, which has also negative repercussions on total factor productivity.

Starting from these considerations, the econometric relationship of the index of total factor productivity (ITFP) is built corresponding to the following restrictions:

– if alpha=0 or =1 (that is when the production would be nonsensical for the labour force or, respectively, for capital), ITFP tends to zero;

– ITFP depends non-linearly on alpha, admitting a maximum when alpha is equal to its long-run (equilibrium) level.

We suggest the simplest functional form for ITFP which would correspond to such restrictions:

$$\mathrm{ITFP} = \underset{(+/-)}{(\mathrm{alpha - alpha^{\wedge}a})} * \underset{(+)}{\mathrm{RV}} \qquad \text{(II.3.2.1)}$$

where RV captures the effect of the rest of variables. The expression (alpha-alpha^a) positively or negatively influences the total factor productivity, depending on the level of alpha.

The first adopted assumption (when alpha=0 or =1, ITFP=0) is automatically observed.

The second one is also satisfied for a>1. The question is: How to determine parameter a?

The long-run (equilibrium) level of alpha is noted alphao. It is estimated separately using a specific procedure. From

$$\partial\mathrm{ITFP}/\partial\mathrm{alpha} = 0 \qquad \text{(II.3.2.2)}$$

we have:

$$1–a*alfao^{\wedge}(a–1) = 0 \qquad \text{(II.3.2.3)}$$

$$1/a = alfao^{\wedge}(a–1) \qquad \text{(II.3.2.4)}$$

$(1/a)^{(1/(a-1))} = alfao$ (II.3.2.5)

3.2b2. Concerning RV, several factors have been retained.

The investment intensity is one of them, because of its decisive role in the technological improvement of the production of goods and services; it is approximated by the gross fixed capital formation in real terms (GFCFc).

The following is the domestic demand pressure (DDP). When this pressure accentuates, an increase of utilisation rate of productive capacities is expectable. A converse tendency can appear in the contrary situation. Normally, such a factor does not instantaneously act; its effect becomes more visible in the next period. Consequently, the first lag is included in econometric specification.

A positive correlation has been also identified between the total factor productivity and unemployment rate,(ru) which probably reflects the pressing influence of the last on labour-intensity of the employed workers.

The influence of transitional reforms is captured by the time (t). A constant is also added in order to reflect the trend of total factor productivity.

3.2c. The following specification is, therefore, adopted:

$$\underset{}{\mathrm{ITFP}} = \mathrm{f}(\underset{(+/-)}{\mathrm{alpha}}, \underset{(+)}{\mathrm{GFCFc}}, \underset{(+)}{\mathrm{DDP(-1)}}, \underset{(+)}{\mathrm{ru(-1)}}, \underset{(+)}{\mathrm{t}}, \mathrm{c}) \qquad \text{(II.3.2.6)}$$

3.3. Special attention has been paid to the private consumption (CH at current prices and CHc at constant ones) as a main component of domestic absorption.

3.3a. As usually, the first explicative factor is the disposable income (YD). Taking into account the available information, this is approximated using data concerning the gross domestic product, general consolidated budget revenues corrected by the government transfers, net incomes and current transfers from abroad.

3.3b. The analysis of the Romanian statistical series revealed a negative correlation between private consumption in real terms, on one hand, and the reference interest rate of NBR (IR), on the other.

3.3c. As other similar works, the present macromodel includes also the previous level of private consumption as an explanatory variable of the current one.

3.3d. Hence, the following relationship has been adopted:

$$\mathrm{CHc} = \mathrm{f}(\underset{(+)}{\mathrm{YDc}}, \underset{(-)}{\mathrm{IR}}, \underset{(+)}{\mathrm{CHc(-1)}}) \qquad \text{(II.3.3.1)}$$

3.4. The public consumption is approximated in relation with the government budget expenditures.

3.5. The gross fixed capital formation (GFCF) is estimated in connection with the disposable income, the reference interest rate of NBR, and the foreign direct and portfolio investment (FDPIE):

$$\mathrm{GFCF} = \mathrm{f}(\underset{(+)}{\mathrm{YD}}, \underset{(-)}{\mathrm{IR}}, \underset{(+)}{\mathrm{FDPIE}}) \qquad \text{(II.3.5.1)}$$

The equations for output and domestic absorption (especially gross capital formation) can be combined obtaining a mini-system of the market equilibrium (IS curve):

3.6. The relationship concerning exports (XGSE) refers to all the transactions with goods and services.

3.6a. Usually, the exports are explained by the foreign demand (regional or world); with this aim, the Romanian macromodel uses the world trade in real terms (WTc).

3.6b. The import (MGSE) is also important. This dependence comes from the fact that the Romanian export industries are based, in a substantial measure, on imported raw materials and energy resources.

3.6c. A significant export determinant is the international competitiveness. It is defined as index (ICOsdr) in dependence on exchange rate, world trade deflator (special drawing rights), and gross domestic product deflator.

The world trade deflator of special drawing rights has been considered more appropriate for the structure of Romanian commercial changes.

The influence of international competitiveness on export increases step-by-step, due to the gradual transition from command to market economy.

3.6d. Consequently, the following estimation is adopted:

$$\underset{}{XGSE} = f(\underset{(+)}{WTc}, \underset{(+)}{MGSE}, \underset{(+)}{ICOsdr}) \qquad (II.3.6.1)$$

3.7. The import is also considered in an extensive meaning (goods and services together).

3.7a. The first its determinant is the domestic absorption. But the main components of this – the final consumption (FCc) and the gross fixed capital formation (GFCFc), both at constant prices – do not have identical influences and, consequently, they are included separately.

3.7b. The international competitiveness is also present. Similarly to export, the effect of this factor on import has a growing trend.

3.7c. As a result, the following specification has been retained:

$$MGSE = f(\underset{(+)}{FCc}, \underset{(+)}{GFCFc}, \underset{(-)}{ICOsdr}) \qquad (II.3.7.1)$$

3.8. The Romanian macromodel admits the gross domestic product deflator (PGDP) as a leading price index. It is derived as the ratio between indices of nominal (IGDP) and real (IGDPc) gross domestic product. It is important to notice that IGDP and IGDPc result from the entire system of behavioural and accounting relationships included in macromodel. In such determination, the gross domestic product deflator seems to be the most representative expression of the supply-demand interaction.

The consumer price index (CPI) and the price index of tangible fixed assets (PK) are, therefore, estimated in two phases:

- first as econometric equations and, subsequently,
- as components of the GDP deflator, with which they must be compatible.

3.9. Regarding the exchange rate (ERE) – besides its sluggishness – two factors are particularly important: the domestic inflation (PGDP) and the foreign capital inflows (NCINXE). The last is interpreted as a sum of exports, net incomes and current transfers, foreign direct and portfolio investment.

The dependence of the exchange rate on its previous level is relatively high. This is probably the consequence of a specific transition circumstance, consisting into initial strong expectation of households and firms for depreciation of local currency. The current inflation plays also a major role. At the same time, there is an increasing influence of the international financial markets. Therefore:

$$ERE = f(\underset{(+)}{ERE(-1)}, \underset{(+)}{PGDP}, \underset{(-)}{NCINXE}) \qquad (II.3.9.1)$$

3.10. The transition processes have progressively enforced the functional role of the monetary variables. Among them, the interest rate holds a particular place. Unfortunately, we had not relevant data concerning the commercial banking system, which developed slower and hesitatingly in Romania. Experience from our previous studies indicates the series of the National Bank’s reference interest rate as the most reliable information.

3.10a. Usually, the interest rate is correlated with inflation and real output. The Romanian macromodel also includes these factors, but not separately. Their cumulative expression – nominal gross domestic product (GDP) – proved more suitable.

3.10b. The connection of interest rate to money supply can be also observed; this factor is expressed by the broad money (M2).

3.10c. The international markets begun to play a more and more important role in the functioning of Romanian economy. Consequently, the short-term interest rate in advanced economies (STIRAE) has been involved in determination of the domestic interest rate.

3.10d. As a result, the relationship

$$\underset{}{IR} = f(\underset{(+)}{IR(-1)}, \underset{(+)}{GDP}, \underset{(-)}{M2}, \underset{(+)}{STIRAE}) \qquad \text{(II.3.10.1)}$$

has been included in macromodel. It allows building a LM curve with a standard slope.

III. THE METHODOLOGICAL FRAMEWORK OF EXERCISE

1. Forecasts in general suffer from various sources of errors. Pindyck and Rubinfeld list four circumstances which might contribute towards errors in models.

1.1. The first possible source of errors comes from an additive term which might be in regressions.

1.2. The second possible source is the estimated coefficients of the behavioural equations which are random variables. Since they are estimated, the values obtained might not be the true values of the coefficients.

1.3. The third possible source of errors comes from the exogenous variables. The values of the exogenous variables have to be chosen for the whole forecasting interval and depending on the author's capability to foreseen future policy decisions the forecasts would be closer or not to the true values of the indicators.

1.4. The fourth source of errors identified by the authors is due to the misspecification of the equations. Any model is a simplification of the reality; the selected equations capture only a fraction of the interactions among economic indicators, and – depending on the author's capabilities – it might be a good or not so good representation of the real world.

2. Keeping in mind the sources of errors, it might be important to check the forecasting properties of the model before it is used. A methodology which can be employed exactly for this purpose is stochastic simulation. It is true however, that stochastic simulations work best for the first three sources of errors, but attempts have been made to use this technique for the fourth source of errors as well (see Fair 1980). The main idea behind stochastic simulation is to generate a set of possible numbers for the econometric coefficients and exogenous variables, depending on which type of error is under investigation, according to some rules (generally, the random number generators are implemented in all statistical softwares) After solving repetitively the model, series of solutions are obtained, the variation of which reveals important functional characteristics of the examined model. This methodology has the property of removing the uncertainty regarding the sensibility of the solutions to the source of the errors which was investigated by the author. Depending on the variation, inferences can be made about the robustness and stability properties of the model under investigation.

3. The same methodology can be applied in order to provide forecasts in the form of confidence intervals for the indicators and the associated probabilities. There are many reasons to admit that such predictions can be more informative – and consequently more useful – for the policy makers. Of course, the difference between the two is only in the aim of the exercise and the presentation of the results. While in the first case the author presents how robust is the model to different types of errors, in the second case the author presents interval of variation for forecasting indicators. In our paper the second issue is the one that we want to investigate.

4. Traversing the literature consecrated to these questions, it is a little surprising how few adepts this methodology has, keeping in mind what a strong instrument it is for the validation of the model or for establishing intervals of variations for the forecasted variables. We found very few papers aimed at investigating model's properties, the majority of them are dealing with the properties of the stochastic

simulation methodology, or its application in different circumstances, and using model testing only as an example of its use.

4.1. The study of McWhorter, Spivey, and Wrobleski (1976) applies stochastic simulation in the context of a Kalman filter with goal to assess the possible consequences of misspecification of the behavioural relationships. In general these coefficients are set arbitrarily, and this is why the paper investigates – using stochastic simulation techniques – the effects of misspecification on the estimated coefficients.

4.2. In a paper by Franz, Goggelmann, Schellhorn and Winker (1998), they applied stochastic simulation to testing robustness of outcomes from policy simulations using a macromodel of the West German Economy. Because of nonlinearities which appear in most macroeconomic models, confidence intervals for the indicators have to be obtained with the help of the stochastic simulation. In addition to presenting the interval estimation, they have tested the robustness of the results, using different approaches to stochastic simulation, including Monte – Carlo techniques. In this paper the authors were interested in estimating the uncertainty coming from errors. Different methods of error generation were compared based on the bias of the estimators, and the ones performing better were the ones using pseudo-random generation algorithms.

4.3. In a paper by Winker (1998), there is outlined the usefulness of stochastic simulations, as opposed to point forecasts, because they provide more information. In addition the author advocated for the use of Quasi – Monte Carlo techniques in order to avoid the problems which are associated with random number generators, namely the fact that they might not be random.

4.4. The procedure of stochastic simulation is relevantly presented by Fair (1993). In this paper, the author describes applying stochastic simulation in order to associate probabilities to different economic outcomes. The methodology that is depicted in this paper is based on the assumption that the distribution of errors/coefficients is normal or it is known. First, the deterministic solution is obtain by solving the model in the usual way, the solution is named the trial version. Additional solutions are obtained by solving the model with errors which are drawn from the normal distribution (or the assumed probability distribution). Based on these results, the expected value of the endogenous variables and their respective variances are computed.

A similar procedure is described when one is interested in analysing the uncertainty related to the coefficients of the behavioural equations. Similarly, coefficients are extracted from a normal distribution (or a known distribution), and the model solved in each case. Finally, the expected values and variance of the indicators is computed.

The third possible source of errors, namely the exogenous variables is more difficult to implement since there is no obvious probability distribution that can be associated with them. They are in general fixed, predetermined on some basis. In this case, there is necessary to make some assumptions for the probability distribution associated with the exogenous variables. One solution is to estimate distributions for the exogenous variables based on past predictions made by the model builders, or other persons. A second approach is to assume for the exogenous variables autoregressive or vector auto-regressive equations, which are then incorporated in the model, and estimated simultaneously with the rest. The methodology is then putted to the test in order to compute event probabilities.

4.5. An interesting paper which uses stochastic simulation techniques for evaluating model's properties was written by Gajda, et al. (1998). Their aim is to assess the properties of the model KOSMOS by computing the expected range of the forecast errors for some important features of this system. The authors were especially interested in studying

- the effects of random disturbances,
- the effects of random variation in equations parameters, and
- the effects generated by error propagation when the forecast for longer intervals of time.

In this paper the model was checked with respect to three issues:

a) the difference between the deterministic forecast (the solution of the model obtained in the usual way) and the mean stochastic forecast,

b) the standard deviation of the stochastic forecast (a large standard deviation is a signal that the model is vulnerable to shocks) and

c) the shape of the distribution of the stochastic forecast.

From a methodological point of view random shocks were induced with the help of a pseudo-random number generator and the shocks were applied to the most important equations of the model. The model was solved for 13 semi-annual periods. Each equation was subjected to two types of errors, the first one was included additively in the equation, which corresponds to non-zero disturbances. The second experiment include besides random disturbances in errors, random shocks to the estimated coefficients with the exception of the intercept and dummy variables. This type of analysis helped identify equations which need to receive more attention in order to increase their forecasting properties.

5. Therefore, regarding the main objectives of stochastic simulations, we can conclude that this methodology was developed in order to check model's properties with respect to different random effects, namely the effects on forecast of random errors, the effects on forecast of random variation in coefficients, and the effects on forecasts of random errors in exogenous variables. The methodology is not very difficult in itself, but it is highly computing intensive. The central idea consists in adding in the equations the random effects that one wants to test, and solve the model repeatedly.

6. Obviously, corresponding to the functional form of the equations included in model, this is linear or non-linear. As it is known, the linear models can be solved in a non-iterative way. after obtaining the reduce forms or the final form (depending on whether the model is static or dynamic). Whenever a model has non-linear equations, its solution is approximated with the help of different numerical methods. Since the most models have non-linear components, they are solved iteratively, and the method is called model simulation. This method of obtaining solutions assumes that there is no uncertainty in the model, the coefficients of the behavioural equations (which are estimates of the true coefficients) are fixed, and the disturbances are assumed to be zero. Thus obtained solution is called deterministic.

Assume we have a model

$$f_i(y_{it}, y_{i(t-1)}, \ldots, x_t \;, \alpha_i)=u_{it} \qquad \text{(III.6.1)}$$

where:
y_{it} – endogenous variables;
x_t – exogenous variables;
α_i – econometric coefficients;
u_{it} – error terms;
$i=1, \ldots, n$, and $t=1,\ldots,T$.

The symbol i is used for the index of equations, from which the first j are behavioural. There is assumed that the vector of error terms

$u_t=(u_{1t}, u_{2t},\ldots, u_{nt})'$ is distributed as a multivariate normal $N(0, \Sigma)$. Technically, checking for model's properties in the presence of random errors is performed as follows. For the specific coefficient vector which is obtained by solving the model, one computes the variance-covariance matrix of the errors Σ. For random error terms, which are extracted from the normal distribution $N(0, \Sigma)$, the model is solved. This solution is a deterministic solution in the case of the given values of the error terms. If the model is solved a sufficient number of times, one obtains a distribution of the indicators which are of interest. From this distribution one can compute means, standard deviation, etc., and asses model's properties.

The methodology differs only slightly if one is interested to check model's properties with respect to random variation of the coefficients. In this case, one needs to compute the covariance matrix of the coefficients. In the case normality is accepted, the distribution of the coefficient vector is $N(\hat{\alpha}, V)$. The model is solved for different coefficients which are drawn from the above distribution, and the values of the endogenous variables are analysed in order to asses model's properties.

There is no rule to indicate how many times the model should be solved in order to have sufficient solutions. Of course, the more solutions we have, the better. But, since the procedure is very time consuming, there is a trade-off in terms of the time lost and the gain in accuracy. In our case we have limited ourselves to 100 solutions of the model, but not all attempts to solve the model were successful (The model converged in around two thirds of the cases). We have not discarded any solution of the model irrespective of how far it was from the deterministic solution, as long as it was feasible from an economic point of view[1].

What we studied in this paper is somewhat different from the outcome of the stochastic simulation. First we are interested in here to provide interval forecasts rather than assess model properties. And second, we have used a slightly different methodology because we believe that the hypothesis of normality is too strong in our case.

7. The bootstrap procedure is an improvement of the stochastic simulation procedure, in the sense that in this case no distributional assumptions need to be made. It is a procedure which is widely used but especially in evaluating estimators from equations, or in the case of small time series models, rather than in the context of model evaluations. Still, it can very easily be applied to these type of exercises.

7.1. We consider again the above model (III.6.1). We have the vector of unknown coefficients $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_m)'$ and u is the matrix of errors for all available periods. In our case we were interested in investigating model's properties under random variation in coefficients of the behavioural equations. The bootstrap procedure draws with replacement from the matrix of the errors a vector, which is used to solve the model and obtained the endogenous variables.

We did not use exactly the bootstrap procedure for testing model's properties. So, what we did was to resample data with replacement and compute 100 estimates $\left(\hat{\alpha}_1^{(j)}, \hat{\alpha}_2^{(j)}, \ldots, \hat{\alpha}_k^{(j)}\right)$ for each behavioural equation. This step did not involve solving the model as a whole, but estimating each behavioural equation of the model with errors added in them. This was done automatically with the help of the STATA software. At the end of step one, we had 43 distributions for the coefficients of the behavioural equations.

7.2. In step two we have used the distributions to generate the coefficients to be used in solving the model. The main problem with our work was the fact that due to the limited number of times we have manually solved the model (100), there is no way to insure the representativity[2] of the coefficients and therefore of the results. For example, if the majority of the coefficients from the 100 solutions of the model come from the middle of their respective distributions, then the conclusions regarding the stability of the indicators of interest could be misleading, in the sense that they could give a rosier picture than the reality might be.

In order to enforce the representativity at least in some degree, we have used an a priori procedure. And the idea behind it was to make sure that the coefficients used to solve the model cover uniformly their distribution computed using the bootstrap procedure, in order to prevent occurrences which are mainly from parts of the coefficient's distribution.

The methodology is quite simple, we have divided the coefficients of the behavioural equations in three groups, and the distributions of the coefficients in three intervals. The total number of coefficients of the behavioural equations is 43. The first group contains of the first 14 coefficients, the second contains the next 14 coefficients, and the last one contains the remaining 15 coefficients. Similarly, the distribution of the coefficients was divided in three intervals, the lower part, the middle part, and the upper part. Table 1 presents the matrix used for building the coefficients for solving the model.

[1] We have discarded solutions which have negative values of certain indicators. For example, the solutions which had negative values for exports or imports etc. were discarded.

[2] The representativity is ensured by the law of large numbers. With the increase in the number of times the model is solved (towards 1000) the distributions of the indicators tend towards the true distribution.

The rule applied in obtaining the vectors of coefficients used for solving the model was that no two groups of coefficients could come from the same part of the distribution. For example, we took G1 from the lower part of the distribution, G2 from the middle part, and G3 from the upper part.

There are six possible combinations that satisfy the above rule:

G1L – G2M – G3U,
G1L – G2U – G3M,
G1M – G2L – G3U,
G1M – G2U – G3L,
G1U – G2L – G3M and
G1U – G2M – G3L,

where the number represents the group of the coefficients and the last letter describes the part of the distribution the coefficient group is coming from.

Table 1 The division of equations in groups

	First Group (G1) C(96) – C(109)	Second Group (G2) C(110) – C(123)	Third Group (G3) C(124) – C(138)
Lower part of the distribution (L)	G1L	G2L	G3L
Middle part of the distribution (M)	G1M	G2M	G3M
Upper part of the distribution (U)	G1U	G2U	G3U

Applying this methodology meant, from a practical point of view, that we divided the empirical distribution of the coefficients (the one obtained using the bootstrapping procedure) in three parts, and mixed each part of it independently from the other ones. We have used a random number generator for mixing the values of the particular part of the distribution, and computed the vector of coefficients by adding all three groups following the given rule. Finally, we have computed a matrix of possible coefficients (from all possible 6 combinations) which were used to solve the model.

7.3. Due to nonlinearities in the behavioural equations, not all vectors of coefficients used for solving the model converged towards economic plausible solutions. If that was the case, we have discarded the coefficients and tried with another set. The rate of success was over 0.5, which meant that from two vectors of coefficients at least one leads to economic viable solution for the indicators.

IV. THE RESULTS OF SIMULATIONS AND THEIR INTERPRETATION

1. Before moving towards discussing the properties of the model we comment briefly the motivation behind using this methodology. As we explained in the previous chapter, the bootstrap procedure was applied to compute distributions for the coefficients of the behavioural equations. Notice that the main difference between the stochastic simulation methodology in the literature and the stochastic simulation procedure we have used consists in the fact that while in the first case some assumptions needed to be made about the distributions of the errors (be it normal or other), in the second case the procedure computes a distribution for the coefficients by applying a bootstrap procedure to the data. The two procedures yield different results only in cases when the distributions fail the normality test.

2. The empirical distributions of the coefficients of the behavioural equations were obtained using bootstrapping. The use of bootstrapping is indicated especially in the case when the hypothesis of normality of the coefficients is not supported by the data. We have tested the distributions with regard to skewness, kurtosis and a joint test of normality that takes into account only these properties of the normal distribution. On the whole, only 6 coefficients out of 43 fail the null hypothesis that the distribution is not normal. The tests are not reported due to length of the article constraints.

The above tests reveal, in our opinion, the necessity of using the bootstrap to compute the distribution of the coefficients. We have performed the bootstrap procedure for all coefficients, irrespective of whether they failed the normality test (skewness or kurtosis) or not.

Having established the methodology for checking model's properties, we move to analysing the results. In our case, we are interested to check the uncertainty in the forecasts coming from the uncertainty in the coefficients of the behavioural equations. We have obtained values of the endogenous variables of interest by solving the model repeatedly. Therefore, in all the simulations, the exogenous variables and the statistical entries do not change (they concern the data for 2005). Only the econometric coefficients of the behavioural relationships vary according to above described assumptions.

4. In order to facilitate the interpretation of the obtained results, Table 2 contains some statistical characteristics of the information presented in the above graphs:

- the mean,
- the standard deviation, and
- the coefficient of variation, defined as the ratio of the standard deviation to the mean.

Taking into account the hypotheses of our simulations, the coefficient of variation maybe considered as a measure of uncertainty in forecasting the respective indicator: the smaller the value, the more credible are the corresponding model estimations, and vice-versa.

Table 2: The mean and standard deviation of the endogenous variables

Indicators	Average	S.D.	Coeff. of variation %
Gross domestic product, crt. pr., bill.RON	280.9704	1.807854	0.6434
Gross value added, crt. pr., bill. RON	246.7803	1.468995	0.5953
Gross domestic product deflator (prev. year=1)	1.138339	0.163974	14.40464
Gross domestic product, const. pr., bill. RON	251.2031	31.31299	12.46521
Final consumption, crt. pr., bill. RON	249.3654	7.611982	3.052541
Gross fixed capital formation, crt. pr., bill. RON	61.94693	3.463432	5.590966
Labour force, mill. Persons	10.36044	1.935378	18.68046
Employment, mill. Persons	9.486004	1.663147	17.53264
Exchange rate, RON per Euro	3.702534	0.910587	24.59362
Exports, crt. pr., bill.Euro	23.89069	7.202075	30.14595
Imports, crt. pr., bill.Euro	32.92878	13.83812	42.02439
Budget revenues, bill. RON	93.67655	0.811577	0.866361
Budget expenditures, bill RON	94.68703	0.609247	0.643432
Consumer price index (previous year=1)	1.14409	0.172738	15.0983
Price index of tang. fixed assets (previous year=1)	1.111685	0.183445	16.50155

Source: Authors' computations.

Analysing Table no.2, we can highlight some properties of the discussed macromodel, derived from the uncertainty in econometric coefficients of the behavioural relationships.

4.1. It is important to notice that the main indicators of volume in current prices – gross domestic product, gross value added, final consumption, gross fixed capital formation, general consolidated budget revenues and general consolidated budget expenditures – have coefficients of variation which are of acceptable magnitude. In our opinion, this comes especially from the fact that, among the exogenous variables included in the macromodel the expected index of disposable income plays a leading role, and it is constant in all the simulations.

4.2. The endogenous variable exchange rate registers a large degree of uncertainty. Its interval of variation ranges from1.5 to 6 RON per Euro. In this case, the behavioural equation used to model the exchange rate is responsible. Either because of the statistical series or/and the adopted specification, the corresponding regression is characterised by high standard errors.

4.3. The stochastic pattern of the exchange rate is transmitted into other indicators, with which it is connected. Among them the most significant are exports, imports, and price indices. These indicators also present relatively great coefficients of variations. In other words, the uncertainty in the exchange rate is transmitted towards other variables as well.

4.4.As expected, the fluctuations of price indices influence the real magnitudes too. In this sense, the gross domestic product in constant prices and employment – which is derived from it – are relevant examples.

5. The same facts can be expressed with the help of the confidence intervals. They are computed using two methods:

- the first method uses the information in the data to construct the confidence intervals. As an example of how the confidence intervals were constructed, consider the 90CI, the lower boundary is the 5-th value and the upper boundary is 95-th value, out of the 100 values for each indicator. The rest of the confidence intervals were constructed similarly.
- the second method assumes the normality of the distributions of the indicators. In order to compute the confidence intervals we have used the appropriate values corresponding to each probability from the normal distribution's table.

The confidence intervals for the selected indicators are displayed in Table 3a (the first method) and Table 3b (the second method).

5.1 Whether one measure works better than the other depends on the shape of the distribution. Of course, in cases in which the distribution is bi-modal confidence intervals build using the first method work better. From Graphs 1 –15 we can form an idea about the shape of the distribution, especially the ones which are bi-modal, but unfortunately, some distributions seem to have more outliers than one would expect to find, for example, the CPI, exports and imports, labour force and employment, to name only a few. This feature might come strictly because the number of points available are too few to be able to estimate the real distribution. But there is also the possibility that the problem comes from the lack of precision of the model. In order to be able to distinguish between the two hypotheses, one would need to significantly increase the number of simulations performed.

Table **3**a. The confidence intervals for the selected indicators (first method)

Indicators	0.95 conf. int.		0.90 conf. int.		0.85 conf. int.		0.80 conf. int.		0.75 conf. int.	
Gross domestic product, crt. pr., bill. RON	277.3	283.2	277.6	282.9	278.5	282.5	278.8	282.3	279.3	281.9
Gross value added, crt.pr., bill. RON	243.6	248.7	243.9	248.4	244.7	248.0	245.0	247.9	245.4	247.5
Gross domestic product deflator (prev. year=1)	1.00	1.45	1.05	1.41	1.06	1.37	1.08	1.34	1.09	1.17
Gross domestic product, const. pr., bill. RON	189.4	273.0	199.1	263.6	203.0	260.8	208.5	258.6	229.5	253.5
Final consumption, current pr., bill. RON	238.6	261.4	242.8	258.1	244.6	257.0	245.3	255.8	248.3	251.5
Gross fixed capital formation, crt pr., bill. RON	57.4	65.70	58.17	64.87	58.45	64.67	59.95	64.54	60.73	64.49
Labour force	7.19	11.92	7.90	11.41	8.13	11.25	8.26	10.85	9.43	9.93
Employment, mill. Persons	6.64	10.92	7.27	10.46	7.51	10.33	7.62	9.91	8.67	9.15
Exchange rate, RON per Euro	2.16	5.58	2.72	5.17	2.94	4.92	2.98	4.67	3.21	4.40
Exports, current pr., bill. Euro	17.31	27.73	19.18	26.46	19.67	26.07	19.94	25.20	21.08	24.02
Imports, current pr., bill. Euro	17.02	49.48	19.55	45.30	20.93	41.70	22.36	38.33	25.03	36.84
Budget revenues, bill. RON	92.28	94.55	92.48	94.47	92.76	94.31	92.92	94.17	93.19	94.08
Budget expenditures, bill RON	93.72	94.88	94.17	94.96	93.52	95.45	93.40	95.12	94.48	94.35
Consumer price index (previous year=1)	0.97	1.48	1.02	1.42	1.03	1.38	1.05	1.35	1.08	1.16
Price index of tang. fixed assets (previous year=1)	0.86	1.48	1.01	1.36	1.04	1.34	1.08	1.31	1.14	1.27

Source: Authors' computations.

Table 3b. The confidence intervals for the selected indicators (second method)

Indicators	0.95 conf. int.		0.90 conf. int.		0.85 conf. int.		0.80 conf. int.		0.75 conf. int.	
Gross domestic product, crt. pr., bill. RON	277.4	284.5	278.0	283.9	278.4	283.6	278.7	283.3	278.9	283.0
Gross value added, crt. pr., bill. RON	243.9	249.7	244.4	249.2	244.7	248.9	244.9	248.7	245.1	247.5
Gross domestic product deflator (prev. year=1)	0.82	1.46	0.87	1.41	0.90	1.37	0.93	1.35	0.95	1.17
Gross domestic product, const. pr., bill. RON	189.8	312.6	199.7	302.7	206.1	296.3	211.1	291.3	215.2	253.5
Final consumption, crt. pr., bill. RON	234.4	264.3	236.8	261.9	238.4	260.3	239.6	259.1	240.6	251.5
Gross fixed capital formation, crt. pr., bill. RON	55.16	68.74	56.25	67.64	56.96	66.93	57.51	66.38	57.96	64.49
Labour force,	6.57	14.15	7.18	13.54	7.57	13.15	7.88	12.84	8.13	9.93
Employment, mill. persons	6.23	12.75	6.75	12.22	7.09	11.88	7.36	11.61	7.57	9.15
Exchange rate, RON per Euro	1.92	5.49	2.20	5.20	2.39	5.01	2.54	4.87	2.66	4.40
Exports, crt. pr., bill. Euro	9.77	38.01	12.04	35.74	13.52	34.26	14.67	33.11	15.61	24.02
Imports, crt. pr., bill. Euro	5.81	60.05	10.17	55.69	13.00	52.86	15.22	50.64	17.01	36.84
Budget revenues, bill. RON	92.09	95.27	92.34	95.01	92.51	94.85	92.64	94.72	92.74	94.08
Budget expenditures, bill RON	93.49	95.88	93.68	95.69	93.81	95.56	93.91	95.47	93.99	94.35
Consumer price index (previous year=1)	0.81	1.48	0.86	1.43	0.90	1.39	0.92	1.37	0.95	1.16
Price index of tang. fixed assets (previous year=1)	0.75	1.47	0.81	1.41	0.85	1.38	0.88	1.35	0.90	1.27

Source: Authors' computations.

5.2. The second method works poorly in the presence of outliers as well. In the case of the exports and imports we can notice very large intervals computed with this method. Generally, it seems that the empirical (first) method works better when the distributions of the indicators are not normal.

Again, when we move to variables that are computed with the help of the exchange rate (including the price indices derived from it), the confidence intervals – calculated by both methods – are extending,

6. In order to identify how important is the distortion introduced by the exchange rate's uncertainty, the same exercise was performed with the exchange rate as an exogenous value. The fixed value was the one obtained when solving the model traditionally. The rest of the coefficients were allowed to change using the same rule as previously. Moreover, for comparability, we have used the same values of the coefficients which were used in the previous exercise.

Table 4 presents the coefficients of variation obtained in both series of simulations. The first of them (described in sections 3–5 of this chapter) is mentioned as a Variant 1, and the second one (with unchanged exchange rate) is noticed as a Variant 2.

Table 4: Comparison of the coefficients of variation obtained in the Variant 1 and the Variant 2 (%)

Indicators	Coeff. of variation in Var. 1	Coeff. of variation in Var. 2
Gross domestic product, crt. pr., bill. RON	0.643	0.149
Gross value added, crt. pr., bill. RON	0.595	0.010
Gross domestic product deflator (prev. year=1)	14.405	14.751
Gross domestic product, const. pr., bill. RON	12.465	12.390
Final consumption, crt. pr., bill. RON	3.053	2.925
Gross fixed capital formation, crt. pr., bill. RON	5.591	5.645
Labour force, mill. persons	18.6805	18.6805
Employment, mill. persons	17.5326	17.5303
Exchange rate, RON per Euro	24.5936	0
Exports, crt. pr., bill. Euro	30.1460	23.9565
Imports, crt. pr., bill. Euro	42.0244	22.7896
Budget revenues, bill. RON	0.8664	0.5136
Budget expenditures, bill RON	0.6434	0.1485
Consumer price index (previous year=1)	15.0983	15.3318
Price index of tang. fixed assets (previous year=1)	16.5016	16.4086

Source: Authors' computations.

Comparatively with Variant 1, the second one characterises by lower coefficients of variations, which confirms above discussed supposition concerning the implications of thr exchange rate equation. Other behavioural relationships could be also submitted to such an analysis.

V. SOME CONCLUDING REMARKS

1. The main aim of our paper was to check the possible advantages of the stochastic simulations on macromodels of emerging market economies (on example of Romania). We think that such a procedure is recommendable, especially in cases, where the business environment is characterised by high uncertainty for a relatively long period.

2. There are at least two reasons – one cognitive, another practical – for which the stochastic simulations are useful.

2.1. On one hand, they can help the researches to investigate the operational properties of a given model and – on this basis – to ameliorate them. Coming back to our example, it is clear that the behavioural relationship for exchange rate has to receive more attention. We do not yet able to establish if its volatility comes from instability of the same statistical series used in regressions, or it is generated by the adopted econometric specification. Both these may be present simultaneously. Supplementary analysis of such problems can substantially improved the macromodel estimations.

2.2. As we already mentioned, the stochastic simulations are important for the policy-makers too. Knowing the confidence intervals of the main forecasted indicators, the authorities – Parliament, Executive Administration, Central Bank – can more efficiently define and promote their desirable targets. The incorporation of uncertainty into decisional mechanism is essential in the context of very dynamic changes that characterise the modern society.

3. Our attempt was dedicated to only one of the sources of uncertainty in macromodel predictions. The future developments of this project will add other such factors. Among them, the uncertainty in exogenous variable seems to have a crucial role. These circumstances can be differentially approached, depending on their possible modifications. The approximation of their limits and of the corresponding probability distributions represent, of course, an extremely complicated research, of high scientific and applicative interest.

4. The stochastic simulations are very intensive from a computational point of view. Taking into account our goal – to illustrate its opportunity for emerging market economies – until now, we limited ourselves to the generally accessible softwears. Undoubtedly, a systematic application of such a procedure needs more specialised and powerful technical tools.

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